Statistical Process Control & Process Capability
What is SPC supposed to tell us?

• When you are doing something you should not
• When you are not doing something you should
• When you are doing things right
How Does SPC tell us?

• Think of SPC as charting, similar to the monitoring (temperature, heart rate, blood pressure) that a doctor and nurse chart on a patient.

• SPC monitors the condition of your job activity for the same reason. When something is not looking ordinary, action is taken.
What good is it to an Operator?

There is a clear identification of when actions are to be taken by empowered operators (i.e., requiring them to identify causes and solve problems).
Why is the Customer interested?

- Tighter control of ‘critical to quality’ attributes (features) of your product will improve their own control.

- Rejects, late deliveries and product returns can disappear.
Why Use Control Charts?

“Failure to use control charts to analyze data is one of the best ways known to mankind to increase costs, waste effort, and lower morale.”

Dr. Donald J. Wheeler
Causes

Common Cause (i.e., Noise)
- Is present in every process
- Is produced by the process itself (the way we do business)
- Can be removed and/or lessened but requires a fundamental change in the process
- Requires Management intervention

Special Cause (i.e., Signals)
- Exists in many operations/processes
- Caused by unique disturbances or a series of them
- Can be removed/lessened by using basic process control to identify opportunities for improvement in our existing process
- Requires Operator intervention
Control Chart Basics

• **Control Chart Components**
  - UCL - Upper Control Limit
  - LCL - Lower Control Limit
  - CL - Center Line (average of dataset)

• **Control charts are designed to be used by operators**
  - Clear and easy to see abnormal conditions
  - Uses the premise that quality is everyone’s responsibility
  - Quality at the Source

• **Processes are in control when variation lies within UCL and LCL**

SPC & Cpₖ
Control Charts

❖ **Purpose**
  ● Separate common cause from special cause variation
  ● Communicates process performance over time

❖ **Limits are statistically calculated based on** $3\sigma$
  ● Determined by the process
  ● Independent of the design or customer specifications

❖ **A process is in control when**
  ● It operates with common (natural) variation
  ● It is not influenced by special cause variation

SPC & $Cp_k$
The Basic Control Chart

Individual / Moving Range

• Use:
  • Basic analysis tool in testing for stability
  • When it's inconvenient or impossible to obtain more than one measurement per sample
  • Or when technology allows for easy measurement of every unit at a minimal cost
  • Data availability is sparse

• Variation:
  • Short Term: Represented by the variation from one unit to the next (MR Chart)
  • Long Term: Represented by a sequence of such events (Individuals Chart)

Charts are based on a Subgroup Size of 1

SPC & Cp_k
SPC & $C_p_k$

Long Term Variation

Short Term Variation

SPC & $C_p_k$
## Control Chart Formulas

<table>
<thead>
<tr>
<th></th>
<th>Upper Limit</th>
<th>Lower Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Individuals</strong></td>
<td>$UCL_X = \bar{X} + \frac{3 \overline{MR}}{d_2}$</td>
<td>$LCL_X = \bar{X} - \frac{3 \overline{MR}}{d_2}$</td>
</tr>
<tr>
<td><strong>Chart Formulas</strong></td>
<td></td>
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<tr>
<td><strong>X-bar Chart</strong></td>
<td>$UCL_{\bar{X}} = \bar{X} + A_2 \overline{R}$</td>
<td>$LCL_{\bar{X}} = \bar{X} - A_2 \overline{R}$</td>
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<tr>
<td><strong>Formulas</strong></td>
<td></td>
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<tr>
<td><strong>MR-Chart</strong></td>
<td>$UCL_{MR} = D_4 \overline{MR}$</td>
<td>$LCL_{MR} = D_3 \overline{MR}$</td>
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<tr>
<td><strong>Formulas</strong></td>
<td></td>
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<tr>
<td><strong>R-Chart</strong></td>
<td>$UCL_R = D_4 \overline{R}$</td>
<td>$LCL_R = D_3 \overline{R}$</td>
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</tbody>
</table>
Interpreting Control Charts

We use the phase “Out of Control” when a control chart rule has been broken. These rules are based on the probability that a chart pattern would occur, if nothing has changed in the process.

This means something unusual has happened –

Question it – Go Check It Out!
8 Rules of Control Charts

1. One (1) point > 3 Standard Deviations from the Center Line (Outside the UCL or LCL {3 Sigma Limit})
2. Nine (9) points in a row on same side of center line
3. Six (6) points in a row, all increasing or all decreasing
4. Fourteen (14) points in a row, alternating up and down
5. Two (2) out of three (3) points > 2 standard deviations from the center line (Same side)
6. Four (4) out of five (5) points > 1 standard deviations from the center line (Same side)
7. Fifteen (15) points in a row within 1 standard deviation of the center line (Either side)
8. Eight (8) points in a row within 1 standard deviation of the center line (Either side)
Detecting Lack of Control

What do you do when you determine that a process is not stable (it’s out of control)?

Check it out!
Process Capability
(% Non-Conforming, Capability Analysis \( \text{Cp} \) & \( \text{Cp}_k \))
Process Capability

- Assesses the relationship between natural variation of a process and design specifications
  - An indication of process performance with respect to upper and lower design specifications

- Application of Process Capability
  - Design products that can be manufactured with existing resources
  - Identify process’ weaknesses
  - Select and qualify new processes
  - Link successive processes to maintain design requirements
Process Capability Illustrated

SPC & Cpk

Cp = 0.5
Cpk = 0.5
DPM = 133,614

σ Quality
Level = 1.5

Cp = 1.0
Cpk = 1.0
DPM = 2,700

σ Quality
Level = 3

Cp = 2.0
Cpk = 1.0
DPM = 1,350

σ Quality
Level = 4

Cp = 2.0
Cpk = 2.0
DPM = .002

σ Quality
Level = 6

Six Sigma (without Shift)

SPC & Cpk
DPM & Quality Levels

Quality Level (sigma)

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<tr>
<th>Off-Centering (sigma)</th>
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<th>4</th>
<th>4.5</th>
<th>5</th>
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<td>32</td>
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<td>12,201</td>
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<tr>
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<tr>
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<td>40,100</td>
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<td>66,800</td>
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<td>6,200</td>
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<td>233</td>
<td>32</td>
</tr>
</tbody>
</table>
Steps for calculating \( Cp, Cpk_u \) & \( Cpk_L \)

- Determine upper and lower spec limits
- Calculate the mean
- Determine or estimate the standard deviation
- Calculate \( Cp \)
- Calculate \( Cpk_u \) & \( Cpk_L \)
- \( Cpk \) should be at least 1.0 or better yet, 1.33
  - An acceptable \( Cpk \) value is dependent upon:
    - How critical the measured process is to downstream processes
    - How a non-conformance will impact on customer satisfaction
    - Industry type must also be considered (i.e. automotive vs. general industry)
- A \( Cpk \) value of 2.0 indicates a 6σ process - a goal we should attempt to attain!

SPC & \( Cpk \)
Calculating Cp & Cpk

**Recall:**
- **Cp**: Process Capability Index
- **Cpk**: Process capability considering location within the upper and lower specification limits or tolerance
- **Cpk** is the smaller of the $C_{pk_U}$ or $C_{pk_L}$

$$C_{pkU} = \frac{USL - \bar{X}}{3\sigma}$$

$$C_p = \frac{USL - LSL}{6\sigma}$$

$$C_{pkL} = \frac{\bar{X} - LSL}{3\sigma}$$
SPC & Cpₖ